

A NEURAL FUZZY GRAPH STUDY OF CONTEMPORARY ROMAN DOMINANCE USING THE STRONG ARC MODEL

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Abstract

This paper offered absolute effective modern Roman dominance in neural fuzzy graphs. For some groups' off-graphs, the right values on the effective RDN are computed. High strong order and degree in the neighbouring arc were used to determine the upper and lower bounds for the entire RDN. The Modern Roman Domination Function [MRDF] does not employ the extra cost for placement at each vertex. As a result, this is regarded as the important process of this MRDF. The existence of roman dominance in neural fuzzy trees is examined. To be a roman dominant set, the set of neural fuzzy cut nodes must meet both a needed and adequate criteria. Additionally, it was discovered that every node of an RDS in a non-trivial f-tree is an event on an f-bridge.

Keywords: RDS, RDN, neural f-tree, neural f-graph, arcs, domination, MRDF, nodes, f-bridge

1. Introduction

The analysis of roman domination is a fascinating issue and a rapidly growing field of graph theory and introduced the analysis of dominating sets in graphs.[1,2] The concepts of stable dominance set and 2-dominating set were introduced in [3]. [4] Started the principle of dominance in f-graphs and calculated multiple DN bounds. In intuitionistic neural fuzzy graphs, [5] introduced the dominating set; dominance numbers, separate set, completecontrolling, and RDN. This study aims on incorporating secure dominance and secures absolute domination in neural fuzzy graphs and motivated by the idea of conquering numbers and their applicability. [6] did some work on neural fuzzy graphs as well. The analysis of dominating sets in graphs began in the 1850s as an issue in the chess game. The issue of deciding the low no. of queens which may be located on a board of chess and here all the squares are occupied or threatened by a queen coin. This was considered as a chess interests in Europe. [8,9] presented the principle of graph dominance in 1962 and investigated it further. [7] Investigates it further. This paper aims to use strong arcs to define complete dominance in neural fuzzy graphs.

The following is a breakdown of the paper's structure. Preliminaries are presented in Section 2, and solid dominance of an f-graph is described in a traditional manner in Section 3. The high RDN of all f-graph and all bipartite f-graph was shown to be 2 times and the neural fuzzy graph's minimum mass of arc (Propositions 3.7,3.8). The order of (Theorem 3.10) is

2. Preliminaries

$dN(v) = \text{PuNS}(v)(u)$, with $\text{NS}(v) = \{u \in V : (u,v) \text{ being a solid arc}\}$. The neural f-graph G 's minimum strong arc neighborhood degree is defined by $N(G) = \min dN(u) : u \in V$, and neural

fuzzy graph G 's maximum strong arc neighborhood degree is defined by
 $1N(G) = \max_{u \in V} dN(u) : u \in V$.

If $(u, v) = (u)(v)$ for all $(u, v) \in V$, a f-graph $G = (\sigma, \mu)$ was analysed to be complete. If $v_1, v_2 \in V_1$ or $v_1, v_2 \in V_2$, the f-graph $G = (\sigma, \mu)$ is said to be divided. G is also known as a complete bipartite neural fuzzy graph if $(u, v) = (u)(v)$ for all $u \in V_1$ and $v \in V_2$. Assume that $G = (\sigma, \mu)$ is a neural fuzzy graph. If - vertex not in D is dominated by exactly one vertex of D , the subset D of V is called a exact DS of G .

In a f-graph, a perfect modern roman DS was used. If any vertex in D is controlled by at least one vertex in (G) , it is said to be a complete perfect DS. The absolute perfect DN is indicated by $\gamma_{pf}(G)$ and is low fuzzy cardinality of the exact roman DS (G) .

3. Roman domination in neural f-graphs:

[8,9] provides a detailed definition of a controlling set which was in the existing literature. Since then, several graph theorists [11,12], among others, have investigated various graph dominance parameters. Refer to [14] for a glossary of roman dominance terms in crisp graphs. If either u or v is a neighbor of u , a vertex dominates another vertex in a graph. However, if one could limit roman dominance such that vertices can only control other vertices if it was a neighbor that would be ideal. A vertex will not be dominant in this situation. As a result, this form of dominance is referred to as open or absolute dominance. If I /, it is mentioned which dominates I directly. This was a vertex that freely controls the vertex in its immediate relative one.

If any vertex is nearby to at least one vertex of A , then the set A of vertices in a graph is an open RDS. As a result, an open dominating set exists in a graph only if and if it may have no separated vertex and the sub-graph γA persuaded by A has no separated vertices. The open domination number J of G was the minimum cardinality of an open dominating set. These ideas prompted researchers to more efficiently reformulate some of the principles in roman dominance.

The important thing about this paper was that it is used solid arcs to achieve absolute dominance in neural fuzzy graphs. The dominance is restricted here so a node can control its powerful neighbors. That was, in / nodes, a node has clear dominance. Since [16]'s parameter, 'absolute dominance number' was dependent upon node mass rather than the mass of arc, this

meaning is needed. The value of the old roman dominance number is reduced when the new term is used, and classic results are extracted in a neural f-graph.

A node in a neural f-graph was considered as roman dominance. [17]. i.e., strongly dominates the nodes in /9: If any node in 4K was a solid neighbour of all K nodes. The set K of nodes is a roman dominant set off.

Definition 3.1.[12]The mass of a RDS K was referred as LK.

$\sum M N$ was the low mass of the strong incident arcs. The low mass of RDS was called RDN of a neural f-graph and it was indicated by or simply. An RDS with a low mass in a neural f-graph G was known as low RDS. Let or stand for the complement of a neural fuzzy graph with a clear dominance number. Now we'll use solid arcs to describe absolute dominance in neural fuzzy graphs.

Definition 3.2.A K set of nodules in a neural f-graph $G: (V, \sigma, \mu)$ was a roman dominating set and each node was a solid neighbour of at least 1 node of K

Statement 3.3.The noted f-graph $G: (V, \sigma, \mu)$ has RDS if and only if have no separated nodes and the induced neural f-graph γ_K have no separated nodes.

Definition 3.4.The mass of RDS K was referred to as LK.

$\sum M N$ was the lowest of the mass of the solid arcs. The minimum weight of RDS of an f-graph G was known as the RDN, which is signified by or simply. A RDS of the complement of a neural f-graph G was its minimum RDS.

minimum weight. Let $\gamma_{st}(\bar{G})$ or $\bar{\gamma}_{st}$ denote the strong total domination number of the G.

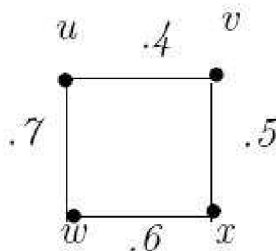


Figure 1: Example of strong domination

Example 3.5. In the fuzzy graph in Figure 1, $(u, w), (w, x), (u, v)$ is a δ arc. Hence $D = \{w, x\}$ is a minimum strong $W(D) = \sum_{u,v \in D} \mu(u, v)$ where $\mu(u, v)$ is the minimum weight

Statement 3.6.

In a non trivial fuzzy graph $G: (V, \sigma, \mu)$ $\gamma_s \leq \gamma_{st}$ a total dominating set is a strong dominating

Proposition 3.7. If $G: (V, \sigma, \mu)$ is a complete fuzzy graph, then $\mu(u, v)$ is the weight of a weakest arc in G .

Proof: Since G is a complete fuzzy graph, all arcs are strong and adjacent to all other nodes. Then the end nodes say $\{u, v\}$ of an

Proposition 3.8. For a complete bipartite fuzzy graph K_{σ_1, σ_2} where $\mu(u, v)$ is the weight of a weakest arc in K_{σ_1, σ_2} .

Proof: In K_{σ_1, σ_2} , all arcs are strong. Also each node in V_1 is adjacent to all nodes in V_2 . Hence in K_{σ_1, σ_2} , the minimum strong total dominating set is one in V_1 and other in V_2 . Then the end nodes say $\{u, v\}$ of an

Remark 3.9. Note that $\min_{v \in V} \sigma(v) < \gamma_{st} \leq p$, since every contains atleast 2 nodes and in a connected fuzzy graph $\gamma_{st} < p$, always.

Theorem 3.10. In a non trivial fuzzy graph $G: (V, \sigma, \mu)$ of order p the following conditions hold.

1. All nodes have same weight.
2. All arcs are M-strong arcs.
3. Each node of G has a unique strong neighbor.

Proof: If all nodes have same weight, all arcs are M-strong and

Theorem 3.11. *In a non trivial fuzzy graph $G: (V, \sigma, \mu)$, if nodes in G is even.*

Proof: Suppose $\gamma_{st} = p$ then by theorem 3.10 each node neighbor, all nodes have same weight and all arcs are M-strong number of nodes say $2n + 1$ then G contains atleast n

Definition 3.12. A neural fuzzy graph's high roman dominant set D If no proper subset of D is a RDS of G , G is said to be a low RDS.

Proposition 3.13. A neural fuzzy graph's minimum solid dominating group was a minimal RDS.

Statement 3.14. Proposition 3.13's inverse does not have to be accurate. The J set was a lowroman dominating group with a mass of 0.8 which was shown in fig 2.

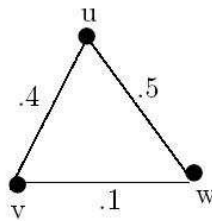


Fig. 2: Example for roman dominating set

Statement 3.15.

In the solid domination case, V was at least set of domination and V/D was the roman set of domination. The illustrations are shown below.

Example 3.16. Figure 3 shows the example of remark.

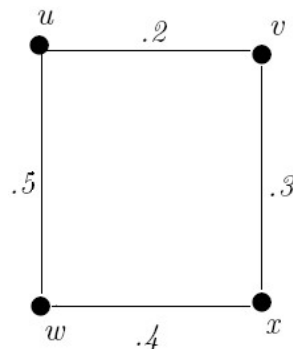


Fig. 3: Example of Remark

In Figure 3, $D = \{u, w, x\}$ is a minimal strong total

Remark 3.17. Let γ and γ_t be the domination number and fuzzy graph respectively defined by A Somasundaram and S. have proved that $\gamma_t + \overline{\gamma_t} \leq 2p$ and equality holds if and only

1. The number of vertices in G is even say $2n$
2. There is a set S_1 of n mutually disjoint effective

Total Domination in Fuzzy Graphs Using Strong

Theorem 3.18. For any fuzzy graph $G: (V, \sigma, \mu)$ without iso

Proof: Since $\gamma_{st} \leq p$, $\overline{\gamma_{st}} \leq p$.

We have $\gamma_{st} + \overline{\gamma_{st}} \leq 2p$

Claim: $\gamma_{st} + \overline{\gamma_{st}} \neq 2p$

Suppose if possible $\gamma_{st} + \overline{\gamma_{st}} = 2p$. Then $\gamma_{st} = p$, $\overline{\gamma_{st}} = p$

Now $\gamma_{st} = p$ implies the number of nodes of G is same weight, every node of G has a unique strong neighbor [Theorem 3.10, 3.11].

If $|\sigma^*| = 2$ then $\gamma_{st} = p$ implies G is complete isolated and this contradicts that $\overline{\gamma_{st}} = p$.

If $|\sigma^*| > 2$ then $|\sigma^*| = 2n$ for some $n > 1$. Since a unique strong neighbor and all arcs are M-strong. In this case

Theorem 3.19. *In any fuzzy graph $G: (V, \sigma, \mu)$ without isola*
 $\gamma_S \leq \gamma_{st} \leq 2\gamma_S$.

Proof: Since every strong total dominating set is a strong dominating set,
 $\gamma_S \leq \gamma_{st}$.

Next to prove $\gamma_{st} \leq 2\gamma_S$.

Let $D = \{v_1, v_2, \dots, v_k\}$ be a minimum strong dominating set. Then the nodes in $V \setminus D$ are therefore openly strong dominated by the nodes in D . For each isolated node v_i , its open strong neighborhood $N_s(v_i)$ is non-empty.

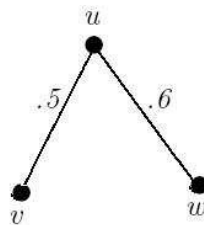


Fig. 4: Neural Fuzzy Graph G

Statement 3.20. The 2 bounds are provided in theorem 3.19. Figure 4 shows the neural fuzzy graph G

In this neural fuzzy graph $\gamma_u = 0.5$. $\gamma_{st} = 0.5 + 0.5 = 1$. 21

Therefore γ_{st} figure 5 consider the neural f-graph

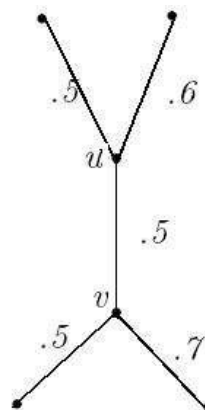


Fig. 5: Example of Theorem 3.20

In this fuzzy graph $D = \{u, v\}$ is both minimum strong total dominating set. Therefore

4. Roman domination in neural f-tree:

A fuzzy sub graph $H: (V, \sigma)$ rotates the neural f-graph $G: (V, \sigma)$ if H is a linked neural f-graph $G: (V, \sigma)$ was known as f-tree. When it has a neural fuzzy rotating sub-graph $F: (V, \sigma)$ then it was a tree for every arcs x and y and not for F . There obtains a way from x to y in F whose power was more than neural f-graph [14]. Here X was a tree which has all nodes and it was considered as a spanning tree. And again note that x was the different MST. An MST of a linked neural f-graph $G: (V, \sigma)$ was a fuzzy rotating sub-graph $T: (V, \sigma)$ like that y was a tree and $\sum_{v \in V} v(\sigma)$ was maximum.

Definition 4.1. [8,14] *An arc is called a fuzzy bridge (f-bridge) if its removal reduces the strength of connectedness between nodes. Similarly a fuzzy cut node (f-cut node) w is a node in G with the property that its removal disconnects the graph.*

Statement 4.2. [11] Every node in an important fuzzy tree G is either a neural fuzzy cut node or a fuzzy end node, and there are at least two neural fuzzy end nodes. An arc is strong in an f-tree if and only if it is an arc of F , where F is the related unique maximum spanning tree. Here there is no 5-solid arcs in a neural fuzzy tree, so these strong arcs are 3-strong. Also keep in mind that an arc in a neural f-tree G is 3 very strong if and only if is an fuzzy bridge of G .

Theorem 4.3. [18] The solid arc incident along with a fuzzy node was a bridge in all unimportant neural fuzzy graph $G: (V, \sigma)$.

Corollary 4.4. [10] The solid neighbour of a neural fuzzy node was a fuzzy cut node G in an important neural fuzzy tree $G: (V, \sigma)$, which excluded K_2 .

Statement 4.5. No 2 fuzzy end nodes are controlled by corollary 4.4. The group of all neural fuzzy nodes in a non-trivial f-tree $G: (V, \sigma)$ except K_2 is never a large roman dominant set.

Theorem 4.6. *In a non trivial fuzzy tree $G: (V, \sigma, \mu)$ except nodes is a strong total dominating set if and only if every fuzzy cut node as strong neighbor.*

Proof: First assume that every fuzzy cut node has at least one neighbor in G . Let D be the set of all fuzzy cut nodes dominating set [11] every node in $V \setminus D$ is openly strong dominated. By our assumption every fuzzy cut node has at least one neighbor. Hence every node in D is strongly dominated by strong total dominating set of G .

Conversely assume that the set D of all fuzzy cut nodes is a strong total dominating set of G . Then every node of G is strongly dominated.

Theorem 4.7. *In a non trivial fuzzy tree $G: (V, \sigma, \mu)$, every strong total dominating set is incident on a fuzzy bridge of G .*

Proof: Let D be a strong total dominating set of G . Let $u \in D$ be a node in the dominating set, there exists $v \in D$ such that (u, v) is a strong edge of the unique MST F of G [7-14]. Hence (u, v) is an f-bridge.

5. The Modern Roman Domination Approach

The authors of [11] used a strict graph-theoretical approach to study the RDP as a variant of the domination issues on graphs, and for the 1st time formalized the Modern Roman Domination Problem. The Roman Empire regions were imagined to be the vertices of a line, with the connections serving as the edges. Each vertex has a label that can be one of three values: 0, 1, or 2. A vertex with the label zero must be together to at least 1 vertex with 2nd label.

Formally, given a graph

$G = (E, V)$, a MRDF was a labelling process $f: V \rightarrow \{0, 1, 2\}$, the following equation was satisfied:

$$\forall u \in V: f(u) = 0 \Rightarrow \exists v \in V: (u, v) \in E \wedge f(v) = 2.$$

The function f makes a partition of $V = (V_0, V_1, V_2)$, where

$$V_i = \{v \in V: f(v) = i\}$$

Let define $n_i = |V_i|$ then have $n_0 + n_1 + n_2 = n = |V|$

The mass f was

$$f(V) = \sum_{u \in V} f(u) = 2 \cdot n_2 + 1 \cdot n_1 + 0 \cdot n_0 = 2 \cdot n_2 + n_1.$$

First, define the MRDF $\gamma_R(G)$ as the low value of a MRDF for every graph G .

$$\gamma_R(G) = \min_{f \in \mathcal{F}} f(V) = \min_{f \in \mathcal{F}}$$

Here \mathcal{F} was the group of all MRDF for G . Many authors are verified the corresponding facts about the MRDF, $\gamma(G)$ and $\gamma_R(G)$

The DN of G :

For all graph $G(V, E)$:

$$\gamma(G), \gamma_R(G) \leq n \quad (1)$$

$$\gamma(G) \leq \gamma_R(G) \leq 2 \cdot \gamma(G) \quad (2)$$

$$\gamma_R(G) = 2 \cdot \gamma(G). \quad (3)$$

$$n_1 = |V_1| = 0, \\ \gamma(G) = \gamma_R(G) = 1 \iff G = K_n, \quad (4)$$

$$\gamma(G) = \gamma_R(G) = n \iff G = K_n, \quad (5)$$

For all graph $G(V, E)$: like that $G[V_1]$ was the sub graph persuaded through V_1 :

$$\Delta(G[V_1])$$

For all graph $G(V, E)$: $u \in V_1, v \in V_2$:

$$(u, v) \notin E. \quad (7)$$

1. The n_1 was minimum and for all graph $G(E, V)$ has no separate vertices.

$$n_0 \geq 3 \cdot n/7. \quad (8)$$

$$\gamma_R(G) \geq \left\lceil \frac{2}{\Delta(G)} \right\rceil$$

For all graph $G(E, V)$ $6 \leq Kn$:

Where $\Delta(G)$ is the maximum 0 of G .

2. By consuming the possibility method, the graph was given $G(E, V)$. And the upper bound was

γ_R :

$$\gamma_R(G) \leq n \cdot \left(2 + \ln \left(\frac{1}{\delta(G)} \right) \right)$$

Where $\delta(G)$ was the low degree of G^1 . It was verified an increased top bound for the MRDF was happened by using the possibility method and inequality $1 - p \leq e^{-p}$:

$$\gamma_R(G) \leq n \cdot \frac{2 \ln(1 + \delta(G))}{1 - e^{-\delta(G)}}$$

Furthermore, the author verified that this top bound was asymptotically best possible, i.e.:

$$\gamma_R(G) \geq n \cdot \frac{2 \ln(1 + \delta(G)) - \ln 4}{1 - e^{-\delta(G)}}$$

The authors verified 2 upper bounds for linked graph G .

1. $\gamma_R \leq 4n/5$. Actually, to be true n has to be ≥ 8 fact if $n = 2$, $\gamma_R = 2 > 4 \cdot n/5 = 8/5$.

Still for any connected graph G with $n \geq 3$, in

1. $|V_0| \geq n/5 + 1$;
2. $|V_1| < 4n/5 - 2$.

The authors of [40] demonstrated some algorithms for computing R in linear T for few graph groups, like co-graphs and interval graphs. It also provided a polynomial procedure for AT-free graphs.

The author of [19] presents a polynomial-time process for calculating the least MRDF for graphs and this denotes that it may also calculate a minimal MRDF for acyclic graphs, since an acyclic graph G may be seen as a forest trees T_i , where T_1, \dots, T_n are single trees, and the RDN of G is the amount of the single Modern Roman Dominating Functions (T_i).

MRD was used to calculate a network problem.

It was used to identify an exact process in linear time remark especially when it is applied to a unit graph.

6. Conclusion:

Domination in graphs is a phenomenon with a lot of theoretical and practical applications. The weighted Roman dominance problem is considered in a connected simple graph, where the cost of positioning at each vertex is added to the costs of possible deployments from a vertex to any of its neighbouring vertices. So here the major role was played by Modern Roman Domination Function [MRDF]. The additional cost for positioning at each vertex is not deployed. Thus this approach is incorporated as a new solution for this issue. In the meantime, 30 and above dominance factors were studied by various scholars. In this research, a new definition known as roman dominance in neural fuzzy graphs is presented. The neural fuzzy equivalent of a well-known finding of the dominance no. and absolute dominance no. of a neuralgraph was demonstrated in this paper. The roman dominance in neural fuzzy trees is also examined in this study. Other clear dominance parameters will be the subject of future articles.

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